$A \land B \Rightarrow C$

T	T	T	T	T
T	F	F	T	T
T	T	T	F	F
T	F	F	T	F
F	F	T	T	T
F	F	F	T	T
F	F	T	T	F
F	F	F	T	F

$(A \Rightarrow C) \lor (B \Rightarrow C)$

T	T	T	T	T	T	T
T	T	T	T	F	T	T
T	F	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	T	T	T	T
F	T	T	T	F	T	T
F	T	F	T	T	F	F
F	T	F	T	F	T	F

The 2 sentences are logically equivalents

$$(P \wedge \neg R) \Rightarrow (Q \Rightarrow R)$$

$$= \neg(P \wedge \neg R) \vee (Q \Rightarrow R)$$

$$= \neg(P \wedge \neg R) \vee (\neg Q \vee R)$$

$$= (\neg P \vee \neg\neg R) \vee (\neg Q \vee R)$$

$$= \neg P \vee \underline{R} \vee \neg Q \vee \underline{R}$$

$$= \neg P \vee \neg Q \vee R$$

$$= \neg(P \wedge Q) \vee R$$

$$= P \wedge Q \Rightarrow R$$ which is a Horn clause

R	$\wedge$	C	$\Rightarrow$	$\neg$	D	$\wedge$	$\neg$	B
T	T	T	F	F	T	F	F	T
T	T	T	F	F	T	F	T	F
T	T	T	F	T	F	F	F	T
T	T	T	T	T	F	T	T	F
T	F	F	T	F	T	F	F	T
T	F	F	T	F	T	F	T	F
T	F	F	T	T	F	F	F	T
T	F	F	T	T	F	T	T	F
F	F	T	T	F	T	F	F	T
F	F	T	T	F	T	F	T	F
F	F	T	T	T	F	F	F	T
F	F	T	T	T	F	T	T	F
F	F	F	T	F	T	F	F	T
F	F	F	T	F	T	F	T	F
F	F	F	T	T	F	F	F	T
F	F	F	T	T	F	T	T	F

$$R \wedge C \Rightarrow \neg O \wedge \neg B$$

$$= \neg(R \wedge C) \vee (\neg O \wedge \neg B)$$

$$= (\neg R \vee \neg C) \vee (\neg O \wedge \neg B)$$

$$= (\neg R \vee \neg C \vee \neg O) \wedge (\neg R \vee \neg C \vee \neg B)$$

$$= (\neg(R \wedge C) \vee \neg O) \wedge (\neg(R \wedge C) \vee \neg B)$$

$$= (R \wedge C \Rightarrow \neg O) \wedge (R \wedge C \Rightarrow \neg B)$$

This is two Horn clauses.

$R \wedge C \Rightarrow (\neg O \wedge \neg B)$ (1)

show that this is not entailed by $R \Rightarrow \neg B$ (2)

Find a model in which sentence 2 is true and sentence 1 is false

R, C, O are true, B is false

$\therefore$ 2 does not entail 1

$$(F \Rightarrow P) \vee (D \Rightarrow P) \Rightarrow ((F \wedge D) \Rightarrow P)$$

LHS is $(F \Rightarrow P) \vee (D \Rightarrow P)$

$= (\neg F \vee P) \vee (\neg D \vee P)$

$= \neg F \vee \neg D \vee P$

RHS is $((F \wedge D) \Rightarrow P)$

$= \neg(F \wedge D) \vee P$

$= \neg F \vee \neg D \vee P$

LHS $\equiv$ RHS so sentence is valid

To prove by resolution, negate and reduce to empty clause

$\neg\,((F \Rightarrow P) \vee (D \Rightarrow P) \Rightarrow ((F \wedge D) \Rightarrow P))$

$= \neg\,(\neg((F \Rightarrow P) \vee (D \Rightarrow P)) \vee ((F \wedge D) \Rightarrow P))$

$= (F \Rightarrow P) \vee (D \Rightarrow P) \wedge \neg((F \wedge D) \Rightarrow P)$

$= (\neg F \vee \neg D \vee P) \wedge \neg(\neg F \vee \neg D \vee P) = (\neg F \vee \neg D \vee P) \wedge (F \wedge D \wedge \neg P$